3.)

$$\dot{W}_{cycle} = \dot{Q}_1 - \dot{Q}_2$$

$$\Rightarrow 40 = \dot{Q}_1 - \frac{1000}{60}$$

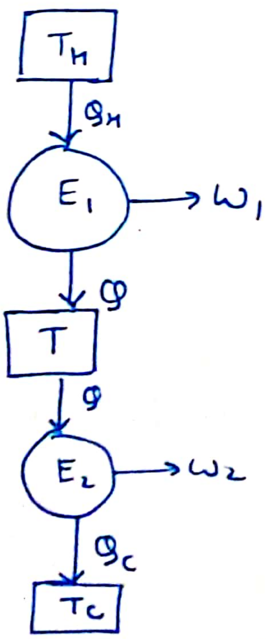
$$\Rightarrow \dot{Q}_1 = 56.67\,kW$$

For a reversible cycle, $\frac{\dot{Q}_1}{\dot{Q}_2} = \frac{T_1}{T_2} \Rightarrow \cancel{\frac{56.67 \times 60}{1000}} = \cancel{\frac{T_1}{200}}$

$$\frac{56.67}{1000} \times 60 = \frac{T_1}{T_2} = \frac{T_1}{200}$$

$$\Rightarrow T_1 = 680.04\,K$$

6.)



(a) net work of 2 cycles are equal.

$$\therefore W_1 = W_2$$

$$\Rightarrow Q_H - Q = Q - Q_C$$

$$\Rightarrow \frac{Q_H}{Q} - 1 = 1 - \frac{Q_C}{Q} \Rightarrow \frac{T_H}{T} - 1 = 1 - \frac{T_C}{T}$$

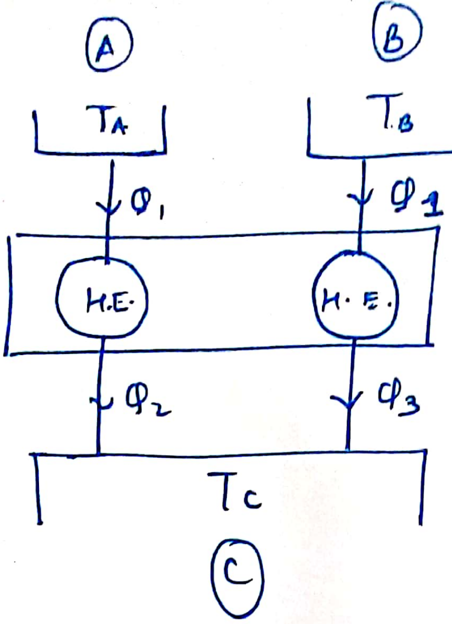
$$\Rightarrow \frac{T_H + T_C}{T} = 2 \Rightarrow T = \frac{T_H + T_C}{2}$$

(b) thermal efficiencies of the 2 cycles are equal.

$$\eta_1 = \eta_2 \Rightarrow 1 - \frac{Q}{Q_H} = 1 - \frac{Q_C}{Q}$$

$$\Rightarrow \frac{Q}{Q_H} = \frac{Q_C}{Q} \Rightarrow \frac{T}{T_H} = \frac{T_C}{T} \Rightarrow T = \sqrt{T_C T_H}.$$

9.)



η of heat engine between (A) & (C)

$$\eta_A = 1 - \frac{T_C}{T_A}.$$

$$\therefore \eta \text{ of our engine} = \alpha \left(1 - \frac{T_C}{T_A}\right) \quad -(1)$$

In our engine,

$$\text{Total heat addition} = 2Q_1 \quad -(2)$$

$$\text{Total heat rejection} = Q_2 + Q_3 \quad -(3)$$

$$\frac{Q_2}{Q_1} = \frac{T_C}{T_A} \Rightarrow Q_2 = Q_1 \frac{T_C}{T_A} \quad -(4)$$

$$\frac{Q_3}{Q_1} = \frac{T_C}{T_B} \rightarrow Q_3 = Q_1 \frac{T_C}{T_B} \quad -(5)$$

Substitute (5) & (4) in (3),

$$\therefore \text{Total heat rejection} = Q_2 + Q_3 = Q_1 T_C \left(\frac{1}{T_A} + \frac{1}{T_B}\right) \quad -(6)$$

Using (2) & (6),

$$\eta \text{ of our engine} = 1 - \frac{Q_2 + Q_3}{2Q_1}$$

$$= 1 - \frac{Q_1 T_C \left(\frac{1}{T_A} + \frac{1}{T_B}\right)}{2Q_1}$$

$$= 1 - \frac{T_C}{2}\left(\frac{1}{T_A} + \frac{1}{T_B}\right) \quad -(7)$$

Equating (1) & (7),

$$\alpha\left(1-\frac{T_C}{T_A}\right) = 1-\frac{T_C}{2}\left(\frac{1}{T_A}+\frac{1}{T_B}\right)$$

Multiply both sides by $\frac{T_A}{T_C}$,

$$\alpha\frac{T_A}{T_C} - \alpha = \frac{T_A}{T_C} - \frac{1}{2} - \frac{1}{2}\frac{T_A}{T_B}$$

$$\therefore \frac{T_A}{2T_B} = \alpha - \frac{1}{2} + \frac{T_A}{T_C} - \alpha\frac{T_A}{T_C}$$

$$= \left(\alpha-\frac{1}{2}\right) + \frac{T_A}{T_C}(1-\alpha)$$

$$\therefore \frac{T_A}{T_B} = (2\alpha-1) + 2(1-\alpha)\frac{T_A}{T_C}$$

2.) Maximum efficiency $= 1-\frac{T_2}{T_1}$

$$= 1-\frac{200}{400} = 0.5$$

$\therefore$ Maximum work output $= 0.5 \times 105\ MJ$

$$= 52.5\ MJ$$

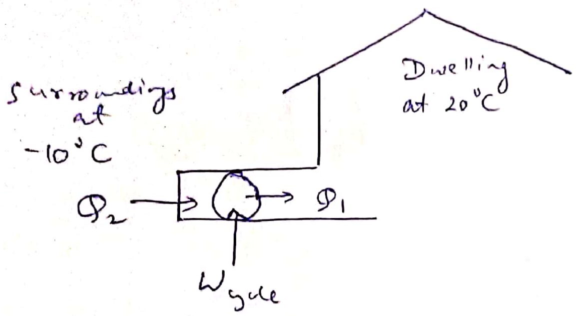
Claim of work, $W = 17.5\ kWh$

$$= 17.5 \times 10^3 \times 3600\ J$$

$$= 63\ MJ > 52.5\ MJ$$

So, one should not invest money to put his engine in the market.

5.)



Heat Pump must provide 3.5×10^6 kJ per day to a dwelling to maintain its temperature at $20^\circ C$.

Electricity cost → Rs 5.0 per kWh

$$COP_{max} = \frac{T_1}{T_1 - T_2} = \frac{293}{293 - 263} = 9.77$$

$$\therefore \dot{W}_{min} = \frac{\dot{Q}_1}{COP} = \frac{3.5 \times 10^6 \text{ kJ}}{9.77} = 0.358 \times 10^6 \text{ kJ per day.}$$

$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

$$\therefore 0.358 \times 10^6 \text{ kJ} = 0.358 \times 10^9 \text{ J} = \frac{0.358 \times 10^9}{3.6 \times 10^6} \text{ kWh}$$

$$= 99.44 \text{ kWh}$$

$\therefore$ In one day, theoretical total operating cost

$= 99.44$ kWh $\times$ Rs 5.0 per kWh $=$ Rs 497.22

8.) Refrigeration cycle – $COP = 3$

Maintains a computer lab at $18^\circ C = 291$ K on a day

when the outside temperature is $30^\circ C = 303$ K.

$$\text{Thermal load} = 30000 \frac{\text{kJ}}{\text{hr}} + 6000 \frac{\text{kJ}}{\text{hr}}$$

$$= 36000 \frac{\text{kJ}}{\text{hr}} = \frac{36000}{3600} \frac{\text{kJ}}{\text{s}}$$

$$= 10 \text{ kW}$$

$$W = \frac{Q_c}{COP} = \frac{10 \text{ kW}}{3} = 3.33 \text{ kW}$$

In case of a reversible refrigerator, COP can be calculated using the absolute hot and cold temperatures.

$$COP_{rev} = \frac{1}{\frac{T}{T_c} - 1} = \frac{1}{\frac{303}{291} - 1}$$

$$= 24.25$$

For the same thermal load, the minimum theoretical power required = power required by a reversible cycle

$$= \frac{q_c}{COP} = \frac{10\ kW}{24.25} = 0.4124\ kW$$

1.) Given $\dot{Q}_2 = 8000$ kJ/hr

COP = 2.5

$$\dot{W} = \frac{\dot{Q}_2}{COP} = \frac{8000}{2.5} = 3200 \text{ kJ/hr}$$

For a reversible cycle

$$COP_{rev} = \frac{T_2}{T_1 - T_2} = \frac{273+0}{293-273} = 13.65$$

$$\Rightarrow \dot{W}_{rev} = \frac{\dot{Q}_2}{13.65} = \frac{8000}{13.65} = 586.1 \text{ kJ/hr}.$$

which is 5½ times less.

4.)) Given, $\eta = 0.29$, COP = 3.5

$$\text{required ratio} = \frac{\text{heat transfer to radiator}}{\text{heat transfer to heat engine}}$$

$$= \frac{Q_c' + Q_H'}{Q_H}.$$

By first law,

$$\underset{HE}{Q_H - Q_c'} = W = \underset{HP}{Q_H' - Q_c}$$

$$\eta = 1 - \frac{Q_c'}{Q_H} \quad \text{(definition for heat engine)}$$

$= 0.29$

$$\frac{Q_c'}{Q_H} = 0.71$$

$$COP = 3.5 \quad \text{(definition for heat pump)}$$
$$= \frac{Q_H'}{Q_H' - Q_c}$$

$$\Rightarrow 1 - \frac{Q_c}{Q_H'} = 1/3.5 = 0.2857$$

$$\frac{Q_c}{Q_H'} = 0.7143$$

Plugging in first law,

$$0.29\,Q_H = Q_H' - Q_c \Rightarrow \frac{0.29\,Q_H}{Q_H'} = 1 - \frac{Q_c}{Q_H'}$$

$$\Rightarrow \frac{Q_H'}{Q_H} = 1.015$$

Therefore required ratio $= \frac{Q_c'}{Q_H} + \frac{Q_H'}{Q_H} = 0.71 + 1.015$
$= \underline{\underline{1.725}}$

7) Given,

$m_A = 20\,kg$ $\quad T_A = 1000\,K$ $\quad W_{max}$? $\quad T_{final}$ = ?

$m_B = 10\,kg$ $\quad T_B = 300\,K$

Engine stops when $T_{Af} = T_{Bf} = T_f$

and maximum work $\Rightarrow$ reversible process

By first law

@ A

$$\delta Q_A - \delta W_A = dU_A \Rightarrow \delta Q_A = dU_A = m_A C_v dT_A$$

@ B

$$\delta Q_B - \delta W_B = dU_B$$

$$\delta Q_B - p\,dV_B = dU_B \Rightarrow \delta Q_B = dU_B + p\,dV_B = dH_B = m_B C_p\, dT_B$$

Reversible $\Rightarrow$ $$\frac{\delta Q_A}{T_A} = \frac{\delta Q_B}{T_B}$$

$$\Rightarrow -\frac{m_A C_v\, dT_A}{T_A} = \frac{m_B C_p\, dT_B}{T_B}$$

because dT_A is −ve and heat transfer to engine is positive

$$\Rightarrow -\frac{dT_A}{T_A} = 0.7\frac{dT_B}{T_B}$$

On integration from respective lower limits to T_f

$$\int_{1000}^{T_f} -\frac{dT_A}{T_A} = 0.7\int_{300}^{T_f} \frac{dT_B}{T_B}$$

$$-\ln(T_f/1000) = 0.7\ln(T_f/300)$$

$$\Rightarrow T_f = 609.11\ \text{K}.$$

By first law

$$\delta W = \delta Q_A - \delta Q_B$$

$$= -m_A C_v\, dT_A - m_B C_B\, dT_B$$

$$= -20 \times 717.5 \times (609.11 - 1000) - 10 \times 1004.5 \times (609.11 - 300)$$

$$= 2504.2615\ \text{kJ}$$